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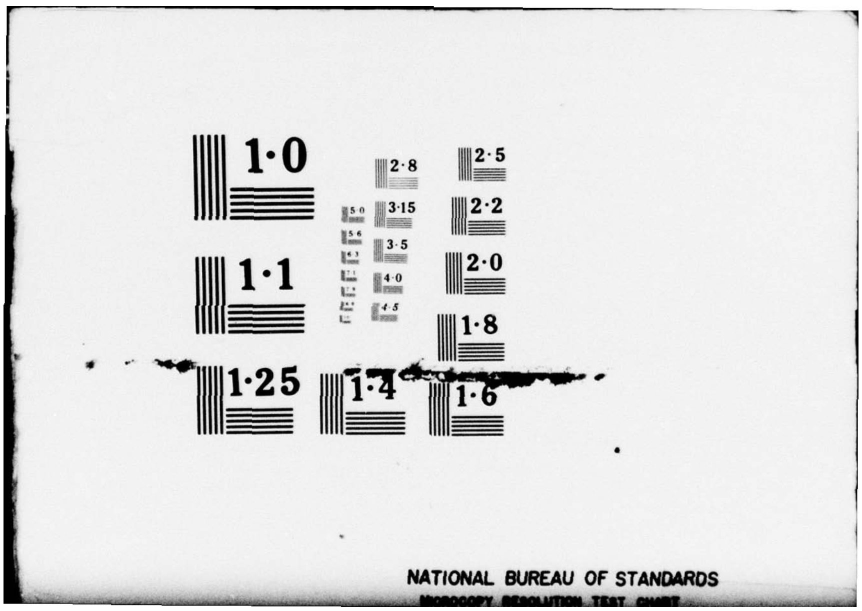
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TR 77122



ROYAL AIRCRAFT ESTABLISHMENT

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Technical Report 77122

August 1977

# A DYNAMIC METHOD OF DETERMINING THE STIFFNESS AND CROSS AXIS STABILITY OF A REPULSION MAGNETIC BEARING

by

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(18) DRIC
(19) BR-60409

UDC 621-218 : 537.8 : 620.177.6

(14) RAE-TR-77122

ROYAL AIRCRAFT ESTABLISHMENT

(9) Technical Report 77122

Received for printing (11) 15 Aug[redacted]977

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(12) 21p.

## SUMMARY

Magnet bearing support systems are becoming of increasing interest in satellite and other engineering projects. In order to design such systems a knowledge of the bearing stiffness is required. This Report analyses the dynamics of a repulsive type magnet bearing and proposes a simple pendulum experiment to determine the radial stiffness by measuring the vibration frequency. The analysis, based on the potential energy of the system, shows the relationship between radial stiffness and the pendulum arm length and also predicts a rotational stability condition, namely that the bearing length to diameter ratio must be greater than $\sqrt{3}$ for stability.

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Departmental Reference: Space 535





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LIST OF CONTENTS

## 1 INTRODUCTION

In recent years there has been increasing interest in the use of magnetic bearings for rotating mechanisms, such as momentum wheels in satellites, in order to obtain a reduction in friction and wear of conventional bearings and hence a greater reliability and lifetime. In order to design such bearings it is necessary to determine the stiffness of the system. One way of doing this is simply to directly measure the forces involved with a static measuring system using force transducers and position measuring equipment. Such a procedure introduces measuring uncertainties due to equipment biases and tends not to give very accurate results. The method is even less accurate when attempting to estimate the cross-axis stability due to the angular stiffness.

An alternative method is proposed here, with the appropriate theoretical analysis, based on a pendulum type experiment. Not only does this method provide the stiffness of a bearing but it can also be used to confirm the 'cockling ratio' which is predicted by the analysis: namely, that a bearing is angularly stable if its length is greater than $\sqrt{3}$ times its diameter. This has been confirmed by the experimental work[1] of Lodge.

The only theoretical analysis[2] that is known to the author is that of Backers'. This work predicted the positional stiffness of a magnet but not the angular stiffness and was limited to a two-dimensional analysis. This latter restriction is removed in the present paper where a full three-dimensional analysis is made.

## 2 DESCRIPTION OF THE EXPERIMENT

The type of repulsion magnetic bearing considered here is illustrated in Fig 1. Such a bearing consists of an outer shell of permanent magnet rings of equal thickness, magnetised axially with the rings arranged so that like poles are adjacent, *ie* alternate rings have opposite directions of magnetisation. Inside this is a coaxial inner shell of magnetic rings of the same thickness and stacked on a shaft in exact correspondence with the outer shell. Such an arrangement is radially stable but axially unstable and the angular stability depends, as mentioned above, on the dimensions of the bearing.

If the shaft of the bearing is now suspended from a point S, Fig 2, on the bearing axis whilst the outer shell is fixed vertically the inner shell is free to oscillate and the frequency of oscillation will be a measure of the forces and couples acting on the bearing. We will now determine what this relationship is.

## 3 THE FIELD OF THE OUTER SHELL

Following Backers[2] we will assume that the intensity of magnetisation M of the magnets follows a sinusoidal spatial distribution axially, *ie*

$$M = \mathcal{M} \sin 2\pi Z/\lambda \tag{1}$$

where $\lambda$ is the periodicity of the system, so that each magnet is of thickness $t = \lambda/2$, and $\mathcal{M}$ is the maximum magnetisation occurring at the centre of a magnet. The Z axis is taken along the bearing axis with origin at the interface between two adjacent magnets. It will also be assumed that the magnets are homogeneous so that the field is radially symmetric and end effects will be ignored.

The magnetic potential $\Omega$ may now be calculated from the formula

$$\Omega = \int \underline{M} \cdot \text{grad} \frac{1}{r} \, dv$$

where the integration is over the outer shell or, integrating by parts,

$$\Omega = -\int \frac{\text{div}\, \underline{M}}{r} \, dv + \int \frac{\underline{M} \cdot d\underline{S}}{r}$$

and the second integral is zero since the surface element $d\underline{S}$ is normal to the magnetisation vector. Consequently, the magnetic potential at a point $Z_0$ along the bearing axis is

$$\Omega(Z_0) = -2\pi \int_{-\infty}^{\infty} dZ \int_{\rho_i}^{\rho_0} \rho d\rho \frac{\mathcal{M}\alpha \cos \alpha Z}{\sqrt{\rho^2 + (Z - Z_0)^2}} \tag{2}$$

where $\rho_i$ and $\rho_0$ denote the inner and outer radii respectively of the outer shell and $\alpha = 2\pi/\lambda = \pi/t$. The integral in equation (2) may be evaluated explicitly to give

$$\Omega(Z_0) = -4\pi\mathcal{M} \cos \alpha Z_0 \left[\rho_i K_1(\alpha\rho_i) - \rho_0 K_1(\alpha\rho_0)\right] \tag{3}$$

where $K_n(x)$ denotes the modified Bessel function of the second kind defined by

$$K_n(x) = \int_0^\infty e^{-x \cosh t} \cosh nt \, dt \quad . \tag{4}$$

Thus the magnetic potential along the bearing axis varies as

$$\Omega(Z) = F \cos \alpha Z \tag{5}$$

where F is a negative constant defined by equation (3) and the suffix 0 has now been omitted. The explicit form for F will not be needed in this Report since it is not necessary to evaluate the absolute forces and couples but only their relative magnitudes.

Now the magnetic potential must satisfy Laplace's equation and consequently its spatial distribution within the region occupied by the inner shell is

$$\Omega(R,Z) = F \cos \alpha Z \, I_0(\alpha R) \tag{6}$$

where R denotes the radial position from the bearing axis and $I_n(x)$ denotes the Bessel function with purely imaginary argument, defined by

$$I_n(x) = \frac{1}{2\pi} \int_0^{2\pi} e^{x \cos \theta} \cos n\theta \, d\theta \quad . \tag{7}$$

Thus the magnetic field $\underline{H} = - \text{grad } \Omega$ is

$$\underline{H} = - \alpha F \sin \alpha Z \, I_0(\alpha R) \hat{\underline{Z}} - \alpha F \cos \alpha Z \, I_1(\alpha R) \hat{\underline{R}} \tag{8}$$

where $\hat{\underline{Z}}$ and $\hat{\underline{R}}$ denote unit vectors in the axial and radial directions respectively. Fig 3 shows the distribution of the magnetic field and potential.

## 4 FORCES AND COUPLES ACTING ON A SINGLE INNER RING

It is shown in the Appendix that for small axial and radial displacements of the centre of an inner ring, a and d respectively, and a small angular

rotation $\theta$ about the equilibrium position the potential energy of a single ring is approximately

$$W = W_0 \left[\cos \alpha a \, I_0(\alpha d) - \frac{\theta^2}{4}\left(\alpha^2 r_0^2 - \frac{\pi^2}{12} + \frac{1}{2}\right)\right]$$

where $r_0$ is the outer radius of an inner ring. Consequently the axial and radial forces $F_a$ and $F_r$ acting on the centre of an inner ring are

$$F_a = -\frac{\partial W}{\partial a} = +\alpha W_0 \sin \alpha a \, I_0(\alpha d)$$

$$F_r = -\frac{\partial W}{\partial d} = -\alpha W_0 \cos \alpha a \, I_1(\alpha d)$$

which approximate for small displacements to

$$F_a \triangleq S_A a$$

$$F_r \triangleq \frac{S_A}{2} d \qquad (9)$$

where $S_A = \alpha^2 W_0$ is the axial stiffness. It is to be noted that this shows that the axial stiffness is twice the radial stiffness $S_R$ independently of any small rotation present.

Similarly, for small displacements, the couple acting about the centre of a ring is

$$C = -\frac{\partial W}{\partial \theta} = +\frac{\theta}{2}\alpha^2 W_0\left(r_0^2 - \frac{t^2}{12} + \frac{t^2}{4\pi^2}\right) = +\theta S_R\left(r_0^2 - \frac{t^2}{12} + \frac{t^2}{4\pi^2}\right). \qquad (10)$$

It is to be noted that this couple has a destabilising influence.

## 5 MOTION OF THE PENDULUM

Let us now consider the total couple acting on the inner shell assembly, about the point of suspension. If there are a total of $n$ inner magnets so that 122

the length of the inner assembly is $\ell = nt$ and $L$ is the distance from the point of suspension to the centre of the inner assembly then the total moment about the point of suspension due to the radial forces acting on each magnet is, using equation (10)

$$M_r = -S_R\theta \sum_{N=1}^{n} [L + (N + \tfrac{1}{2} - n)t]^2$$

$$= -S_R\theta n\left(L^2 + \frac{n^2 - 1}{12} t^2\right) .$$

The moment due to the axial force is second order in $\theta$ since the axial displacement depends upon $1 - \cos\theta$ and may thus be neglected to first order in $\theta$. Adding now the destabilising couple of each magnet gives a total moment about the point of suspension

$$M_s = M_r + nC$$

$$= -S_R\theta n\left[L^2 + \frac{1}{12}\left(\ell^2 - 3D^2 - \frac{3t^2}{\pi^2}\right)\right] \qquad (11)$$

where $D = 2r_0$ is the diameter of an inner ring.

From this equation it can be seen that if there is a long suspension arm, then the moment about the point of suspension is equal to the arm length $L$ multiplied by the total radial stiffness $S_{RT} = nS_R$ of the bearing, *ie* the destabling couples may be ignored. On the other hand, when the inner bearing is supported about its centre (or freely suspended with the aid of a stabilising axial servo), the moment about its centre depends upon the quantity $\ell^2 - 3D^2 - (3t^2/\pi^2)$. Since the term $3t^2/\pi^2$ is negligible, the bearing will thus be rotationally stable if its length is greater than $\sqrt{3}$ times its diameter and unstable otherwise.

If, now, $m$ is the mass of the inner bearing and $k$ is its radius of gyration about an axis through its centre normal to the bearing axis, then the total moment of inertia about the point of suspension is



$$I = m(L^2 + k^2) + I_{arm} \qquad (12)$$

where $I_{arm}$ is the moment of inertia of the supporting pendulum shaft. Also, if $-\theta M_g$ is the moment about the point of suspension due to gravitational forces, the total moment is $M_s - \theta M_g$ and consequently the oscillation frequency of the pendulum, $\omega$, in rad/s is given by

$$\omega^2 = \frac{M_s + M_g}{I} = \frac{S_{RT}\left[L^2 + \frac{1}{12}\left(\ell^2 - 3D^2 - \frac{3t^2}{\pi^2}\right)\right] + M_g}{m(L^2 + k^2) + I_{arm}} \quad . \qquad (13)$$

The gravitational term will be positive if the point of suspension is above the pendulum, otherwise negative, and is proportional to the arm length $L$. For practical bearings this term will be insignificant compared with that contributed from the radial stiffness. For long arm lengths, then, the oscillation frequency will approach the asymptotic value

$$\omega_a = \sqrt{\frac{S_{RT}}{m}} \qquad (14)$$

independently of the arm length. Consequently the period of oscillation provides a measure of the radial stiffness of the bearing and, by varying the arm length, the rotational stability condition can be investigated. Fig 5 shows the predicted variation of $\omega^2$ with $L^2$.

## 6 CONCLUSION

The analysis has shown the relationship between the radial stiffness, the geometry of the bearing suspension system and the frequency of oscillation, equation (13). By measuring the period of oscillation for long arm lengths a direct measure of the radial stiffness is available. Furthermore by varying the arm length the rotational stability condition can be investigated. A limited number of experiments have been reported in Ref 1 and support the predictions of this Report.

## Appendix

## POTENTIAL ENERGY OF A SINGLE RING

Consider the situation where the inner shell is rotated by a small angle $\theta$ about the point of suspension S, Fig 4. Let this be in the plane XZ with the Y axis forming a right hand set, and let a and d denote corresponding axial and radial displacements of the Nth inner magnet. We will now introduce another cartesian frame of reference (xyz) with its origin at the centre of the magnet, z axis along the axis of the magnet and the xz plane coinciding with the XZ plane. Thus if (x,y,z) are the coordinates of a point in the magnet, then the corresponding (X,Y,Z) coordinates are

$$\begin{aligned} X &= R\cos\Phi = d + x\cos\theta + z\sin\theta \\ Y &= R\sin\Phi = y \\ Z &= (N - \tfrac{1}{2})t + a - x\sin\theta + z\cos\theta \end{aligned} \tag{A-1}$$

where $(R,\Phi,Z)$ are the corresponding cylindrical coordinates.

Now the potential energy W of the ring is given by the equation

$$W = -\mu_0 \int \underline{H} \cdot \underline{M}_i dv \tag{A-2}$$

where $\underline{H}$ is the magnetic field of the outer shell, $\underline{M}_i$ is the magnetisation of the inner ring and the integration is over the volume of the inner ring. Now

$$\begin{aligned} \underline{M}_i(z) &= \mathscr{M}\sin\left[\alpha(N - \tfrac{1}{2})t + z\right]\hat{\underline{z}} \\ &= (-)^{N+1}\mathscr{M}\cos\alpha z\,\hat{\underline{z}} \end{aligned} \tag{A-3}$$

where $\hat{\underline{z}}$ is a unit vector along the z axis. Consequently, by integrating equation (A-2) by parts and noting that $\Omega = 0$ at the z boundaries, the potential energy is

$$W = -\mu_0 \int \Omega \frac{\partial M_i}{\partial z} dv$$

and thus, using equations (A-1), (A-3) and (6),

$$W = -\mu_0 \int dv\alpha F\mathscr{M} \sin \alpha z \sin \alpha \,(a - x \sin \theta + z \cos \theta) I_0(\alpha R) \quad .$$

Introducing polar coordinates $(r,\psi)$ , so that $x = r \cos \psi$ , $y = r \sin \psi$ , and the integral representation (7) for $I_0$ gives

$$W = -\mu_0 \alpha F\mathscr{M} \int r dr d\psi dz \sin \alpha z \sin \alpha \,(a - r \cos \psi \sin \theta + z \cos \theta)$$

$$\times \frac{1}{2\pi} \int_0^{2\pi} e^{\alpha R \cos (t - \Phi)} \, dt \quad . \qquad \text{(A-4)}$$

For convenience we will now use the thickness of a magnet as the unit of length so that we may put $\alpha = 1$. Using equation (A-1) equation (A-4) may then be put in the form

$$W = -\mu_0 F\mathscr{M} \int_{r_i}^{r_0} r dv \int_{-\pi/2}^{\pi/2} dz \sin z \operatorname{Im} J \qquad \text{(A-5)}$$

where Im J denotes the imaginary part of the integral

$$J = \frac{1}{2\pi} \int_0^{2\pi} dt \int_0^{2\pi} d\psi \; e^{i(a + z \cos \theta - r \cos \psi \sin \theta)}$$

$$\times \; e^{(d + r \cos \psi \cos \theta + z \sin \theta) \cos t + r \sin \psi \sin t}$$

and $r_0$ and $r_i$ denote the inner and outer radii of the inner shell.

Expanding J to second order in $\theta$ about $\theta = 0$ produces

$$2\pi J = \int_0^{2\pi} dt \int_0^{2\pi} d\psi \, e^{i(a+z) + d\cos t + r\cos(\psi - t)}$$

$$\times \Bigg[1 - \theta(ir\cos\psi - z\cos t)$$

$$- \frac{\theta^2}{2}\left(iz + r\cos\psi\cos t + 2irz\cos\psi\cos t + r^2\cos^2\psi - z^2\cos^2 t\right)\Bigg]$$

and using equation (7) to evaluate the integrals results in

$$\frac{J}{2\pi} = e^{i(a+z)}\Bigg[I_0(r)I_0(d) - \theta I_1(d)\left(irI_1(r) - zI_0(d)\right)$$

$$- \frac{\theta^2}{2}\left(izI_0(r)I_0(d) + r(1 + 2iz)I_1(r)\,\frac{I_0(d)+I_2(d)}{2}\right)\Bigg] . \qquad \text{(A-6)}$$

Now consider the $z$ integration occurring in equation (A-5) for $W$. Firstly, the term independent of rotation in equation (A-6) contributes a potential energy

$$W_1 = -2\pi\mu_0 F\mathcal{M}\int_{r_i}^{r_0} r\,dr \int_{-\pi/2}^{\pi/2} dz \sin z \sin(a+z) I_0(r) I_0(d)$$

$$= W_0 \cos a \, I_0(d) \qquad \text{(A-7)}$$

where $W_0$ is the equilibrium potential energy of a ring

$$W_0 = -\pi^2\mu_0 F\mathcal{M}\left[r_0 I_1(r_0) - r_i I_1(r_i)\right] . \qquad \text{(A-8)}$$

The remaining terms in equation (A-6) are already of second order for $d$ small and hence, to second order, we may put a zero, *ie* the rotational energy is independent of small axial motion. Furthermore, with $a = 0$, the $\theta$ coefficient occurring in equation (A-6) gives zero contribution to $W$ since the integrand of the $z$ integration in equation (A-5) is an odd function of $z$. From this we may conclude that for small translation and rotation about the

equilibrium position, the nett forces acting at the centre of an inner magnet are independent of rotation and the couple is independent of translation.

The remaining $\theta^2$ coefficient in equation (A-6) contributes, for $a$ and $d$ small, a rotational energy

$$W_2 = \pi \frac{\theta^2}{2} \mu_0 F \mathcal{M} \int_{r_i}^{r_0} r\,dr \int_{-\pi/2}^{\pi/2} dz \sin z \left[\sin z \left(rI_1(r) + (r^2 - z^2) I_0(r)\right) + 2z \cos z \left(I_0(r) + rI_1(r)\right)\right] . \quad \text{(A-9)}$$

Using the basic integrals

$$\int rI_0 dr = rI_1 , \qquad \int r^2 I_1 = r^2 I_2 , \qquad \int r^3 I_0 = r^3 I_1 - 2r^2 I_2 ,$$

$$\int_{-\pi/2}^{\pi/2} \sin^2 x\, dx = \frac{\pi}{2} , \qquad \int_{-\pi/2}^{\pi/2} x \sin 2x\, dx = \frac{\pi}{2} , \qquad \int_{-\pi/2}^{\pi/2} x^2 \sin^2 x\, dx = \frac{\pi}{4}\left(\frac{\pi^2}{6} + 1\right)$$

reduces equation (A-9) to

$$W_2 = \frac{\theta^2}{4} \pi^2 \mu_0 F \mathcal{M} \left[\left(r^2 - \frac{\pi^2}{12} + \frac{1}{2}\right) rI_1(r)\right]_{r_i}^{r_0} . \quad \text{(A-10)}$$

Now provided that $(r_0 - r_i)$ is several times the thickness of a magnet, as is always the case in practice, the contribution from $r_i$ in equation (A-10) is negligible. Consequently, the rotational energy may be approximated by

$$W_2 \simeq \frac{\theta^2}{4} \pi^2 \mu_0 F \mathcal{M} r_0 I_1(r_0) \cdot \left(r_0^2 - \frac{\pi^2}{12} + \frac{1}{2}\right)$$
$$\simeq -\frac{\theta^2}{4} W_0 \left(r_0^2 - \frac{\pi^2}{12} + \frac{1}{2}\right)$$

using equation (A-8).

Summarising, then, and re-introducing the factor $\alpha$ the potential energy of a displaced inner ring is given approximately by

$$W = W_0 \left[\cos \alpha a \; I_0(\alpha d) - \frac{\theta^2}{4}\left(\alpha^2 r_0^2 - \frac{\pi^2}{12} + \frac{1}{2}\right)\right]$$

for small displacements $a$ , $d$ and a small rotation $\theta$ .

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| No. | Author | Title, etc |
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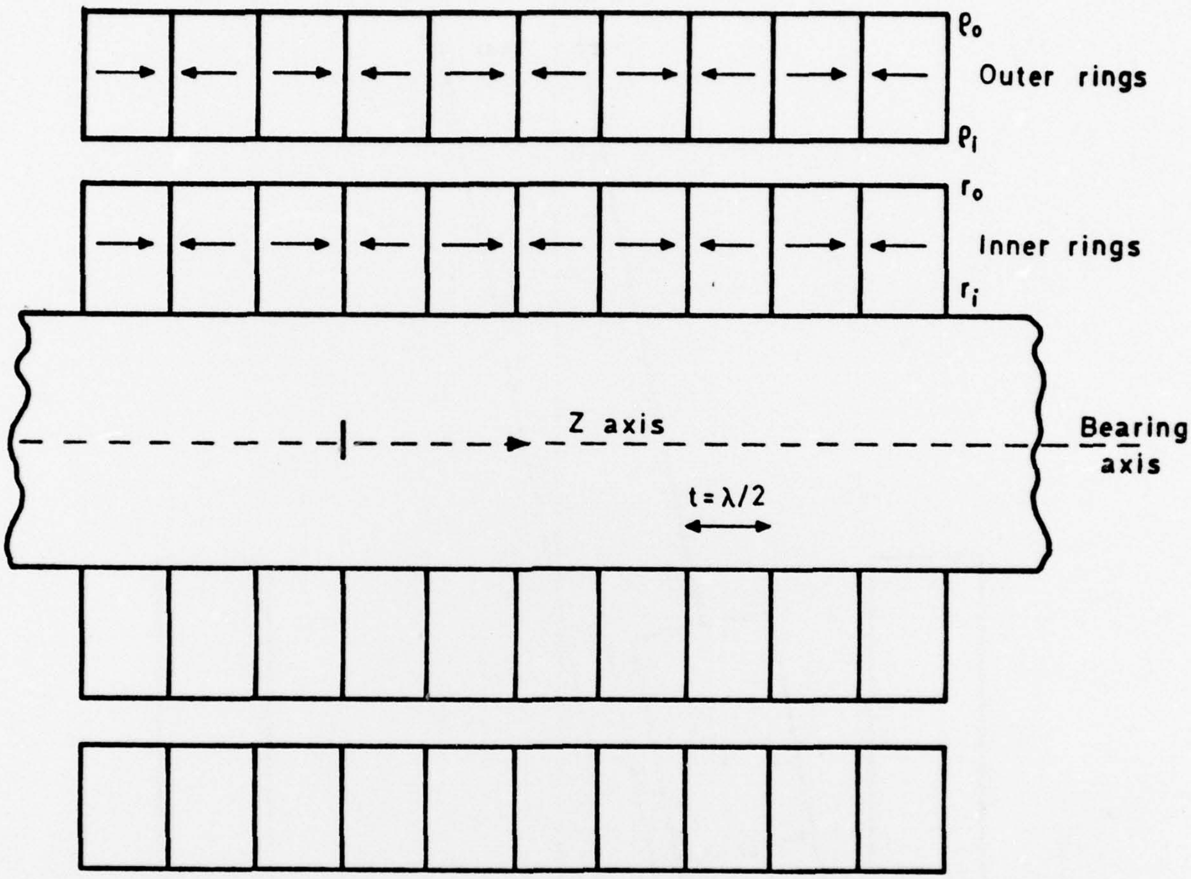
| 2 | F.T. Backers | A magnetic journal bearing. Phillips Technical Review 22, 232-238 (1961) |




Arrows indicate direction of magnetisation in each ring

Fig 1 Cross section through repulsion type magnetic bearing

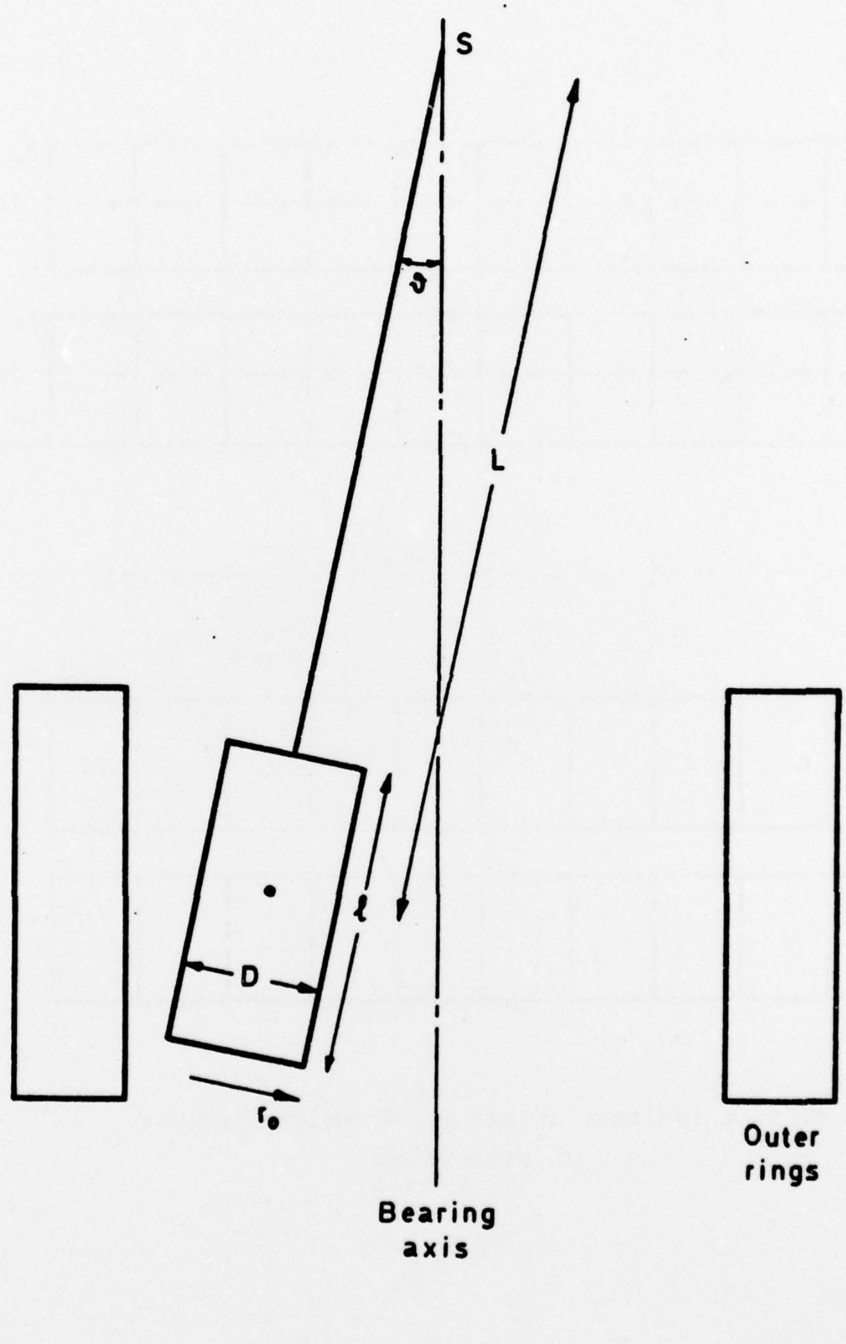



Fig 2 Pendulum experiment

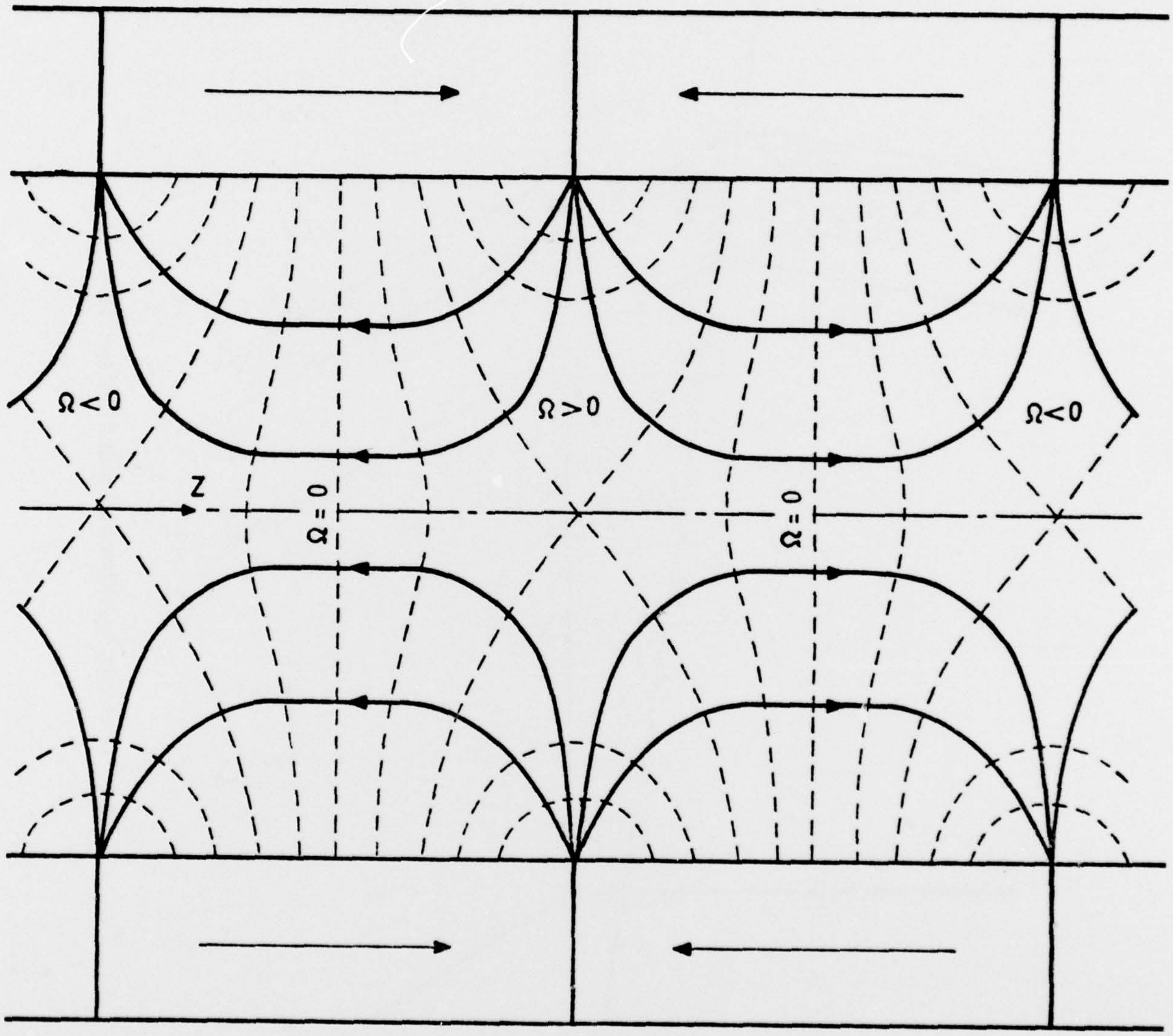



Fig 3 Magnetic field and potential of outer shell

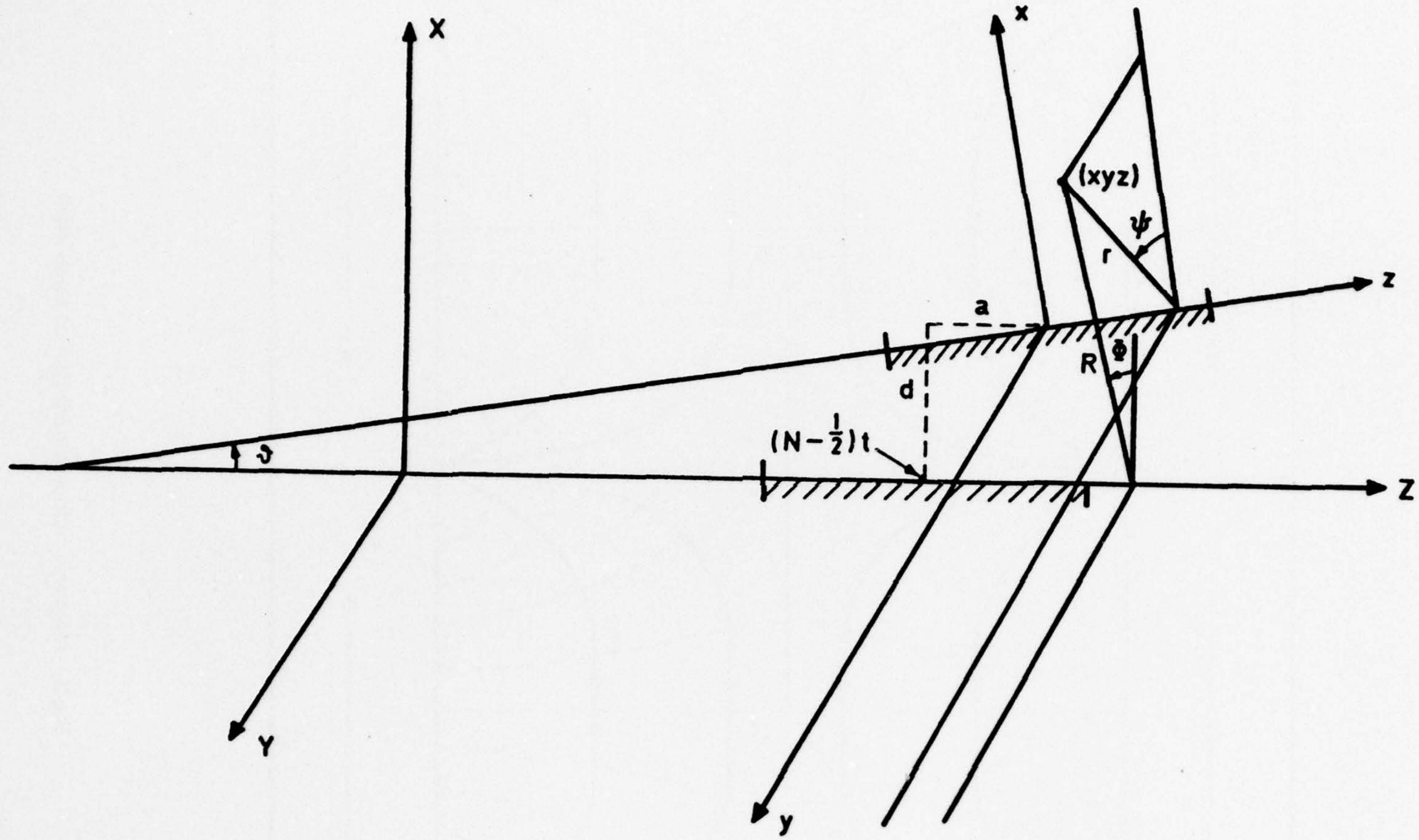



Fig 4 Coordinate systems of rotation and translation

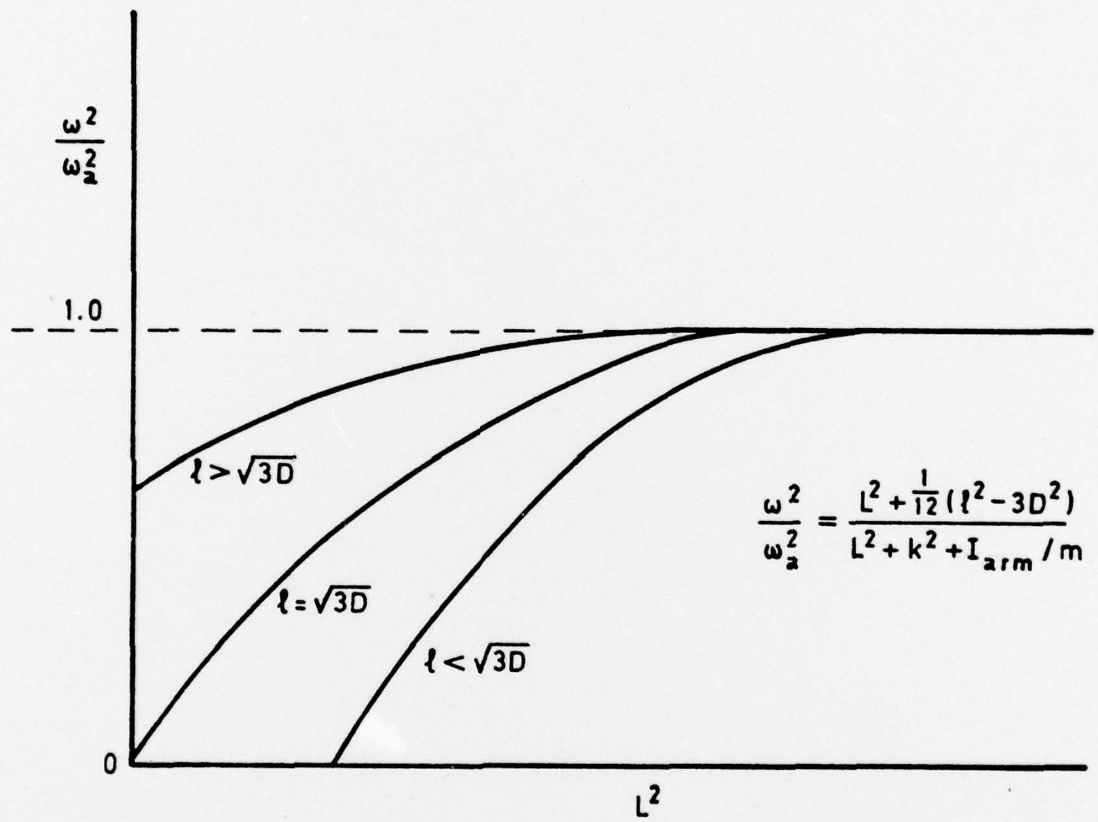



Fig 5 Variation of oscillation frequency with arm length

## REPORT DOCUMENTATION PAGE

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| 1. DRIC Reference (to be added by DRIC) | 2. Originator's Reference | 3. Agency Reference | 4. Report Security Classification/Marking |
|---|---|---|---|
| | RAE TR 77122 | N/A | UNCLASSIFIED UNLIMITED |

| 5. DRIC Code for Originator | 6. Originator (Corporate Author) Name and Location |
|---|---|
| 850100 | Royal Aircraft Establishment, Farnborough, Hants, UK |
| **5a. Sponsoring Agency's Code** | **6a. Sponsoring Agency (Contract Authority) Name and Location** |
| N/A | N/A |

7. Title A dynamic method of determining the stiffness and cross axis stability of a repulsion magnetic bearing

7a. (For Translations) Title in Foreign Language

7b. (For Conference Papers) Title, Place and Date of Conference

| 8. Author 1. Surname, Initials | 9a. Author 2 | 9b. Authors 3, 4 .... | 10. Date | Pages | Refs. |
|---|---|---|---|---|---|
| Plimmer, R.N.A. | | | August 1977 | 19 | 2 |

| 11. Contract Number | 12. Period | 13. Project | 14. Other Reference Nos. |
|---|---|---|---|
| N/A | N/A | | Space 535 |

15. Distribution statement

(a) Controlled by – [redacted]

(b) Special limitations (if any) –

16. Descriptors (Keywords) (Descriptors marked * are selected from TEST)

Magnetic bearings. Cross axis stiffness.

17. Abstract

Magnet bearing support systems are becoming of increasing interest in satellite and other engineering projects. In order to design such systems a knowledge of the bearing stiffness is required. This Report analyses the dynamics of a repulsive type magnet bearing and proposes a simple pendulum experiment to determine the radial stiffness by measuring the vibration frequency. The analysis, based on the potential energy of the system, shows the relationship between radial stiffness and the pendulum arm length and also predicts a rotational stability condition, namely that the bearing length to diameter ratio must be greater than $\sqrt{3}$ for stability.

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